1. If $f\left(x+\frac{1}{x}\right) = x^2+\frac{1}{x^2},\ x \neq 0$, then $f(x)$ equals
   A) x^2+1 B) x^2+2 C) x^2-1 D) x^2-2

2. The sum of $\log_4 2 - \log_8 2 + \log_{16} 2 - \ldots$ is
   A) $\log_e 2$ B) $\log_e 2 - 1$ C) $1 - \log_e 2$ D) $1 + \log_e 2$

3. Let $f(x) = \begin{cases} x, & if\ x \in \mathbb{Q} \\ \sin x, & if\ x \in \mathbb{R}-\mathbb{Q} \end{cases}$,
   where $\mathbb{Q}$ is the set of rationals and $\mathbb{R}$ is the set of real numbers
   Then the value of $f'(0)$ is
   A) 1 B) -1 C) 0 D) $\frac{\pi}{4}$

4. Let (X, *d*) be a metric space. Consider now the following statements:
   1. The empty set φ and the whole set X are closed sets.
   2. The union of an infinite number of closed subsets of X is closed.
   3. The intersection of an infinite number of closed subsets of X is closed.

   Which of the above statements is/are true?

   A) 1 and 2 only B) 1 and 3 only
   C) 2 and 3 only D) 1, 2 and 3

5. Consider the following statements on vectors of R^3:
   1. The vectors (1, -3, 7), (2, 0, -6), (3, -1, -1), (2, 4, -5) are linearly dependent.
   2. The vectors (1, -2, 1), (2,1,-1), (7, -4,1) are linearly independent.

   Which of the above statements is/are true?
   A) 1 only B) 2 only
   C) Both 1 and 2 D) Neither 1 nor 2

6. Given that the set $S = \{e^t, e^{2t}, te^{2t}\}$ is a basis of a vector space V of functions $f : R \to R$. Let D be the differential operator on V defined by $D(f) = \frac{df}{dt}$. Then the matrix representation of D relative to the basis S is:

   A) $\begin{bmatrix}1&0&0\\0&2&1\\0&0&2\end{bmatrix}$ B) $\begin{bmatrix}1&0&0\\0&2&0\\0&1&2\end{bmatrix}$ C) $\begin{bmatrix}1&0&0\\0&2&2\\0&1&2\end{bmatrix}$ D) $\begin{bmatrix}1&0&0\\0&2&2\\0&0&2\end{bmatrix}$

7. Let $A = \begin{bmatrix}3&2&1\\0&2&0\\0&0&-1\end{bmatrix}$ and $B = \begin{bmatrix}1&2&1\\0&1&4\\0&0&-2\end{bmatrix}$.

   Then what is the value of the determinant of AB?

   A) 24 B) 16 C) 8 D) 12

8. Let A and B be two square matrices of order 5 each and let $\rho(M)$ denote the rank of a matrix M. If $\rho(A) = 5$ and $\rho(B) = 4$, then what is the minimum value of $\rho(AB)$?

A) 1 B) 2 C) 3 D) 4

9. Let A^+ be used to denote the Moore-Penrose g-inverse of a matrix A. Then which of the following is true?
A) A^+ exists always and is not unique
B) A^+ exists always and is unique
C) A^+ may not exist always and is unique if it exists
D) A^+ is defined only when A is square matrix

10. Consider the following statements for limit superior and limit inferior of a sequence $\{A_n\}$ of sets:
1. limit inf A_n is the set of points belonging to all but finitely many of the sets A_n
2. limit sup A_n is the set of points belonging to infinitely many of the sets A_n
3. limit inf $A_n \subset$ limit sup A_n

Which of the above statements is/are true?

A) 1 and 2 only B) 1 and 3 only
C) 2 and 3 only D) 1, 2 and 3

11. Which one of the following statements about the Borel field $\mathbb{B}$ of subsets of $\mathbb{R}$ is not true?
A) $\mathbb{B}$ is a σ-field generated by the class $\{(-\infty, x), x \in \mathbb{R}\}$
B) $\mathbb{B}$ is a minimal σ-field containing the class $\{(-\infty, x), x \in \mathbb{R}\}$
C) $\mathbb{B}$ is a field generated by the class $\{(-\infty, x], x \in \mathbb{R}\}$
D) $\mathbb{B}$ is a σ-field generated by the class $\{(-\infty, x], x \in \mathbb{R}\}$

12. Let $\{f_n\}$ be a sequence of non-negative measurable functions and let f be a measurable function such that $f_n \to f$ in measure (μ). Then which of the following is true?

A) $\int f d\mu \leq \lim\inf \int f_n \, d\mu$
B) $\int f d\mu \geq \lim\inf \int f_n \, d\mu$
C) $\int f d\mu \geq \lim\sup \int f_n \, d\mu$
D) $\int f d\mu \leq \lim\inf \int f_n \, d\mu$ only for the case $\int f d\mu < \infty$

13. A fair coin is tossed 5 times. What is the probability that the number of heads exceeds the number of tails?
A) $\frac{1}{2}$ B) $\frac{3}{4}$ C) $\frac{21}{32}$ D) $\frac{7}{32}$

14. If $P(A) = 0.9$ and $P(B) = 0.8$, then the minimum value of $P(A \cap B)$ is
A) 0.9 B) 0.8 C) 0.72 D) 0.7

15. Suppose that an urn contains 5 white and 4 blue bulbs. Two bulbs are drawn at random from the urn without replacement. Then probability that the drawn bulbs are of both colors is:

A) $\frac{2}{9}$ B) $\frac{1}{9}$ C) $\frac{7}{9}$ D) $\frac{5}{9}$

16. Person A speaks truth in 75 % cases and another person B speaks truth in 80 % cases. Then the probability that they contradict each other is:

A) $\frac{3}{20}$ B) $\frac{13}{20}$ C) $\frac{7}{20}$ D) $\frac{17}{20}$

17. Consider families with two children and assume that all possible distributions of gender are equally likely. Let *E* be the event that a randomly chosen family has atmost one boy and *F*, the event that the family has both genders. Then which of the following is true?

A) *E* and *F* are independent
B) *E* and *F* are not independent
C) *E* and *F* are mutually exclusive
D) *E* and *F* are independent and mutually exclusive

18. Let $X : \Omega \to R$ be a random variable. Let $\mathbb{A}$ be a σ-field of subsets of Ω and $\mathbb{B}$, the Borel field of subsets of $\mathbb{R}$. Which of the following statements is not true?

A) $X^{-1}(B) \in \mathbb{A}$ for some $B \in \mathbb{B}$
B) $X^{-1}(B) \in \mathbb{A}, \forall B \in \mathbb{B}$
C) $X^{-1}(-\infty, x) \in \mathbb{A}, \forall x \in \mathbb{R}$
D) $X^{-1}(\mathbb{B}) \in \mathbb{A}$

19. Let X, Y be two random variables such that Cov $(X, Y) = 0$. Then we can conclude that

A) X, Y are independent B) Cov $(X^2, Y^2) = 0$
C) Cov $(X^3, Y^3) = 0$ D) $V(X + Y) = V(X) + V(Y)$

20. Let the p.d.f. of a random variable X be $f(x) = \begin{cases} \frac{4-x}{16}, & -2 < x < 2 \\ 0, & otherwise \end{cases}$

Then $P\left(|X| < 1\right)$ is:

A) 0 B) ¼ C) ½ D) ¾

21. If X is a non-negative random variable with c.d.f. *F* and if *E(X)* exists, then the expression for *E(X)* is

A) $\int_0^\infty [F(x) - 1]dx$ B) $\int_0^\infty F(x)dx$

C) $\int_0^\infty [1 + F(x)]dx$ D) $\int_0^\infty [1 - F(x)]dx$

22. Let $P(s)$ be the probability generating function of a nonnegative integer valued random variable X. Then $\int_0^1 P(s)ds$ equals

A) $E\left(\frac{1}{X+1}\right)$ B) $E\left(\frac{1}{X}\right)$ C) $E\left(\frac{1}{X-1}\right)$ D) $E(X)$

23. The characteristic function of a random variable which is degenerate at 0 is

A) e^{-t} B) e^{it} C) e^{t} D) 1

24. Let $\{X_n, n \geq 1\}$ be a sequence of random variables defined on a probability space (Ω, A, P) and X be a random variable, defined on the same probability space. Now, consider the following relations among convergence of $\{X_n\}$:

(1) $X_n \xrightarrow{P} X \Longrightarrow X_n \xrightarrow{d} X$ (2) $X_n \xrightarrow{r} X \Longrightarrow X_n \xrightarrow{P} X$ (3) $X_n \xrightarrow{a.s.} X \Longrightarrow X_n \xrightarrow{P} X$

Which of the above relations is/are true?

A) 1 and 2 only B) 1 and 3 only C) 2 and 3 only D) 1 , 2 and 3

25. Let $\{X_n\}$ be a sequence of i.i.d. random variables and for $n \geq 1$, let $S_n = \sum_{k=1}^{n} X_k$.

Then $\frac{S_n}{n} \xrightarrow{a.s.} \mu$ if and only if $E\left|X_1\right| < \infty$, where $\mu = E(X_1)$. This law of large numbers is known as:

A) Kolmogorov's B) Khintchin's
C) Chebychev's D) Bernoulli's

26. Name the version of the central limit theorem given below:

Let $\{X_n\}$ be a sequence of i.i.d. random variables for which $E(X_1^2) < \infty$ and let

$S_n = \sum_{k=1}^{n} X_k, n \geq 1$. If $E(X_1) = \mu$ and $V(X_1) = \sigma^2 > 0$, then $\frac{S_n - n\mu}{\sqrt{n}\sigma} \xrightarrow{d} Z$, where $Z \sim N(0,1)$.

A) Lindberg-Levy B) Demoivre- Laplace
C) Lindberg-Feller D) Liapunouv

27. Let X have binomial distribution with mean 10. Then which one of the following is a possibility for the variance of X?

A) 14 B) 12 C) 4 D) 6

28. Given that random variable X has a Poisson distribution with parameter value 1. Then value of $E[1/(X + 1)]$ is:

A) 1 - (1/e) B) 1/e C) 1/2 D) 1

29. Consider the hyper geometric distribution based on M items of first kind and $N — M$ items of second kind, from which n items are drawn without replacement at random. If X denotes the number of items of the first kind in the sample, then $E(X)$ is

A) $\frac{nM}{N}$ B) $\frac{n}{N}$ C) $\frac{M}{N}$ D) $\frac{nM}{N-M}$

30. Suppose that X is a random variable with p.d.f. given by $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & otherwise. \end{cases}$

Given that 0.04, 0.36, 0.0225, 0.0625 and 0.49 are five randomly drawn numbers in the interval (0,1). Using the above random numbers, which of the following can be considered a random sample drawn from the above distribution?

A) 0.04, 0.36, 0.0225, 0.0625, 0.49
B) 0.08, 0.72, 0.045, 0.125, 0.98
C) 0.02, 0.18, 0.01125, 0.03125, 0.245
D) 0.2, 0.6, 0.15, 0.25, 0.7

31. If X follows a beta distribution of the first kind with parameters α and β, denoted by $\beta_1(\alpha, \beta)$, then the distribution of $1-X$ is
A) $\beta_1(\beta, \alpha)$
B) $\beta_1(\alpha, \beta)$
C) gamma with parameters with α and β
D) gamma with parameters with β and α

32. Distribution of a sample mean is the same as that of each component random variable in the sample. Then the distribution can be
A) Cauchy
B) Normal
C) Discrete distribution taking nonnegative integers
D) Exponential

33. Let Z_1 and Z_2 be independent standard normal variates. Consider now the following statements:
1. The distribution of $(Z_1 - Z_2)$ is the standard normal.

2. The distribution of (Z_1/Z_2) is the standard Cauchy.

3. The distribution of $(Z_1^2 + Z_2^2)$ is $X^2_{(2)}$

Which of the above statements is/are true?

A) 1 and 2 only
B) 1 and 3 only
C) 2 and 3 only
D) 1, 2 and 3

34. If $X_1, X_2, \dots . X_n$ is a random sample of size n arising from $U(0,1)$, then the distribution of the first order statistic $X_{1:n}$ is
A) uniform over (0,1)
B) standard Cauchy
C) beta distribution of the second kind with parameters $(n, 1)$
D) beta distribution of the first kind with parameters $(1, n)$

35. Let $X_{1:n}, X_{2:n}, \ldots . X_{n:n}$ be the order statistics of n independent random variables with common p.d.f., $f(x) = e^{-x}, x \geq 0$. Then the distribution of $W = X_{r+1:n} - X_{r:n}$ is

A) exponential with mean 1 B) exponential with mean $\frac{1}{n-r}$

C) exponential with mean $\frac{1}{r}$ D) exponential with mean r

36. If X is the standard normal random variable, then the distribution of X^2 is

A) exponential with parameter value 1/2
B) gamma with parameter values (1/2, 1/2)
C) Cauchy with parameter values (1/2, 1/2)
D) normal with mean 0 and variance 1

37. If X_1, X_2, X_3 are independent standard normal variates, then the distribution of

$Y = \frac{\sqrt{2}\,X_1}{\sqrt{X_2^2 + X_3^2}}$ is

A) Student's t with 1 d.f. B) standard normal
C) Student's t with 2 d.f. D) F with (1, 2) degrees of freedom

38. Given $P\{F_{10,12} > 4.30\} = 0.05$, where $F_{10,12}$ is F −statistic with 10 and 12 degrees of freedom. Then the value of $P\{F_{12,10} < 0.2326\}$ is

A) 0.50 B) 0.01 C) 0.05 D) 0.95

39. Let $(X, Y) \sim$ bivariate normal $\left(\mu_1, \mu_2, \sigma_1^2, \rho^2\right)$ Then the conditional distribution of $X/Y = y$ is

A) univariate $N\left(\mu_1 + \frac{\sigma_1}{\sigma_2}(y - \mu_2),\ \sigma_1^2\rho^2\right)$
B) univariate $N\left(\mu_1 + \frac{\sigma_1}{\sigma_2}(y - \mu_2),\ \sigma_1^2(1 - \rho^2)\right)$
C) univariate $N\left(\mu_1 + \rho\frac{\sigma_1}{\sigma_2}(y - \mu_2),\ \sigma_1^2\rho^2\right)$
D) univariate $N\left(\mu_1 + \rho\frac{\sigma_1}{\sigma_2}(y - \mu_2),\ \sigma_1^2(1 - \rho^2)\right)$

40. Consider the following statements on some bivariate distributions:
1. There is only one bivariate normal distribution for which both marginal distributions are univariate normal distributions.
2. There are many bivariate exponential distributions for which marginal distributions are univariate exponential distributions.
Which of the above statements is/are true?

A) 1 only B) 2 only
C) Both 1 and 2 D) Neither 1 nor 2

41. Let $(X_1, X_2, \dots . X_n)$ be a random sample from $U(0, \theta)$ distribution, where $\theta > 0$. Consider the following statements:

1. The nth order statistic $X_{n:n}$ is unbiased for θ.
2. $X_{n:n}$ is consistent for θ.

Then which of the above statements is/are true?

A) 1 only B) 2 only C) Both 1 and 2 D) Neither 1 nor 2

42. If $X_1, X_2, \dots., X_n$ are i.i.d. random variables with common distribution with p.d.f

$f(x, \theta) = e^{-(x-\theta)}$, $x > \theta$; $\theta \in \mathbb{R}$, then UMVUE of θ is

A) $X_{1:n} - \frac{1}{n}$ B) $X_{1:n}$ C) $X_{n:n}$ D) $X_{n:n} + \frac{1}{n}$

43. If $X_1, X_2, \dots . X_n$ are i.i.d. random variables with common $N(\theta, 1)$ distribution, where $\theta \in \mathbb{R}$, then an ancillary statistic for θ is given by

A) $X_{1:n}$ B) $X_{n:n}$
C) $\bar{X}$ D) $\frac{1}{(n-1)} \sum_{i=1}^{n} (X_i - \bar{X})^2$

44. Let X have the Bernoulli distribution with parameter θ, $0 \le \theta \le 1$ and let $h(\theta) = \theta(1-\theta)$ be a parametric function. If $X_1, X_2, \dots, X_n$ are independent and identically distributed as Bernoulli distribution, then the maximum likelihood estimator (m.l.e.) of $h(\theta)$ is given by

A) X B) $\bar{X}$ (1− $\bar{X}$) C) $(\bar{X})^2$ D) $\bar{X}$ ($\bar{X}$ − 1)

45. Let $X_1, X_2, \dots., X_n$ be a random sample from$U[\,\theta - \frac{1}{\theta} + \frac{1}{2}.]$. Then an m.l.e. of θ is

A) $\frac{X_{n:n} + X_{1:n}}{2}$ B) $\frac{X_{n:n} - X_{1:n}}{2}$ C) $X_{1:n} - \frac{1}{2}$ D) $X_{n:n} + \frac{1}{2}$

46. Which of the following statements about the moment estimators is not true?

1. Moment estimators are asymptotically unbiased.
2. Moment estimators are generally consistent.
3. Moment estimators are always efficient.
4. Moment estimators are asymptotically normal under certain conditions.

A) 1 B) 2 C) 3 D) 4

47. A sample of size 1 is taken from an exponential distribution with mean θ. For testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$, the test to be used is given by

$$\phi(x) = \begin{cases} 1, if x > 2 \\ 0, otherwise \end{cases}$$

Then power of the test is given by

A) e^{-2} B) e^{-1} C) $\frac{e^{-4}}{2}$ D) $\frac{e^{-2}}{4}$

48. Consider the following statements for likelihood ratio test:
1. For testing $\underline{\theta} \in \ominus_0$ against $\underline{\theta} \in \ominus_1$, the likelihood ratio test is a function of every sufficient statistic for θ.
2. Under some regularity conditions, $-2\ln \lambda(\underline{X})$ under H_0 is asymptotically distributed as chi-square.

Which of the above statements is/are true?

A) 1 only B) 2 only C) Both 1 and 2 D) Neither 1 nor 2

49. To test $H_0 : M = M_0$, where M is the median of a distribution function $F(x)$ and M_0 is a specified value. Assume that $F(x)$ is continuous in the vicinity of M. Let $\theta = P\{X > M_0\} = P\{X < M_0\}$ and let K be the test statistic. Then the null distribution of K is

A) Poisson (θ) B) Binomial (n, θ)
C) Bernoulli (θ) D) Geometric (θ)

50. Observed values in two independent samples from two continuous distributions with distribution functions F and G are as follows:

X: 2, 3, 5, 7, 9, 11, 16
Y: 4, 6, 8, 10, 12, 13, 14, 17

The value of the Mann-Whitney test statistic U for testing $H_0 : F(x) = G(x), \forall x$ is given by

A) 40 B) 39 C) 37 D) 41

51. Consider the following statements for the sequential probability ratio test (SPRT):
1. The sample size is fixed.
2. SPRT is intended to test a simple null hypothesis against a composite alternate hypothesis.

Which of the above statements is/are true?

A) 1 only B) 2 only C) both 1 and 2 D) neither 1 nor 2

52. Let $X_1, X_2, \ldots, X_n$ be i.i.d. $N(\mu, \sigma^2)$ random variables, where μ is unknown. Then $(1 - \alpha)$-level confidence interval for σ^2 is given by

A) $\left(\frac{(n-1)S^2}{X^2_{n-1,a/2}}, \frac{(n-1)S^2}{X^2_{n-1,\ 1-a/2}}\right)$ B) $\left(\frac{nS^2}{X^2_{n,\ a/2}}, \frac{nS^2}{X^2_{n-1,\ a/2}}\right)$

C) $\left(\frac{S^2}{X^2_{n,a/2}}, \frac{S^2}{X^2_{n,\ 1-a/2}}\right)$ D) $\left(\frac{S^2}{X^2_{n-1,a/2}}, \frac{S^2}{X^2_{n-1,\ 1-a/2}}\right)$

53. If the loss function is the absolute error, $L(\theta, \delta) = |\theta - \delta|$, then the Bayes' estimator of a real parameter θ is given by
A) mean of the posterior distribution of θ
B) median of the posterior distribution of θ
C) median of the prior distribution of θ
D) mode of the posterior distribution of θ

54. Consider the following statements regarding consistency of an estimator T_n of a parameter θ:
1. T_n is consistent for θ if $T_n \xrightarrow{P} \theta$.
2. If T_n is consistent for θ, then $E(T_n) \longrightarrow \theta$ and $V(T_n) \longrightarrow 0$ as $n \longrightarrow \infty$.
3. Consistency is a large sample property
Which of the above statements is/are true?

A) 1 and 2 only B) 1 and 3 only
C) 2 and 3 only D) 1, 2 and 3

55. From a population of size 10, a sample of size 3 is drawn. Then the total number of possible samples to be drawn from the population using SRSWR exceeds the total number of samples using SRSWOR by a number equal to
A) 880 B) 520 C) 120 D) 50

56. Let a population of size $N = 5$ have its mean $\bar{X}_N = 12$ and $S^2 = 100$. A simple random sample of size $n = 2$ is drawn without replacement. If the sample mean is $\bar{X}_n$, then $E(\bar{X}_n{}^2)$ is:
A) 30 B) 50 C) 144 D) 174

57. If a stratified sample of size 45 is to be selected by Neymann allocation from a population with $N_1 = 150$, $N_2 = 350$, $S_1^2 = 4$, $S_2^2 = 9$, then the number of units to be selected from the first stratum is:
A) 20 B) 10 C) 35 D) 75

58. From a population of size 20, a sample of size 4 is to be selected by systematic sampling. If the 3rd unit is first selected, then what are the other units?
A) 7, 15, 20 B) 5, 9, 13 C) 18, 13, 8 D) 7, 15, 18

59. It is given that $N = 1000$, $n = 100$, $\bar{x} = 250$, $\bar{y} = 500$, $\bar{X} = 275$. Then the ratio estimator of $\bar{Y}$ is

A) 625 B) 575 C) 550 D) 525

60. A sample of 2 units is to be selected from a population of N units as described below. In the first draw one unit is drawn from the population with probability proportional to its size and in the next draw, one unit is drawn by simple random sampling from the remaining $(N-1)$ population units. If the selection probability of a particular unit in the first draw is p, then the probability that this unit is included in the sample is given by

A) $\frac{(N-1)p}{N-2} + \frac{1}{N-1}$ B) $\frac{(N-2)p}{N-1} + \frac{1}{N-1}$

C) $p + \frac{1}{N-1}$ D) $\frac{p}{N-1}$

61. Consider the probability proportional to size sampling scheme without replacement in which two units U_1 and U_2 are drawn at the first and second draws respectively from a population of N units. Let y_1 and y_2 be the characteristic values of U_1 and U_2 with respective initial probabilities of selection p_1 and p_2. Then which of the following estimators is not an unbiased estimator of the population mean?

A) $\frac{y_1}{Np_1}$ B) $\frac{1}{N}\left[y_1 + y_2\frac{(1-p_1)}{p_2}\right]$

C) $\frac{1}{2N}\left[\frac{(1-p_1)}{p_1}y_1 + \frac{(1+p_1)}{p_2}y_2\right]$ D) $\frac{1}{2N}\left[\frac{(1+p_1)}{p_1}y_1 + \frac{(1-p_1)}{p_2}y_2\right]$

62. Let Y_1, Y_2, Y_3, Y_4 be four independent random observations with means

$E(Y_1) = \theta_1 + \theta_2 = E(Y_3)$, $E(Y_2) = \theta_1 + \theta_3 = E(Y_4)$ and common variance σ^2. Consider the following statements:

1. $\theta_1 - \theta_2$ is estimable.
2. $\theta_1 + 2\theta_2 - \theta_3$ is estimable.

Which of the above statements is/are true?

A) 1 only B) 2 only C) Both 1 and 2 D) Neither 1 nor 2

63. Consider the following statements on linear combinations of observations Y_1, Y_2, Y_3.

1. $Y_1 - 2Y_2 + Y_3$ is a linear contrast.
2. The linear contrasts $Y_1 + Y_2 - 2Y_3$ and $2Y_1 - 2Y_2$ are orthogonal.

A) 1 only B) 2 only C) Both 1 and 2 D) Neither 1 nor 2

64. Which of the following principles of experimentation is/are used in the completely randomized design?

A) Local control only B) Replication only
C) Randomization only D) Both replication and randomization

65. In a RBD with r blocks, t treatments and one missing observation, the degrees of freedom of the error sum of squares is

A) $rt - r - t$
B) $rt - r - 1$
C) $rt - r - t + 1$
D) $rt - r - t - 1$

66. In a Latin square design, what is an unbiased estimate of σ^2?

A) row mean square
B) column mean square
C) treatment mean square
D) error mean square

67. For a 2^3-factorial design with r replications, what is the error df?

A) 8
B) 6
C) 8(r-1)
D) 6(r - 1)

68. Match List I with List II and select the correct answer using the codes given below the lists.

List I	List II
a. No local control	1. Analysis of covariance
b. Elimination of variation caused by concomitant variables	2. Incomplete Block Design
c. Handling of largeness of the no. of treatment combinations	3. CRD
d. Controlling of heterogeneity of the experimental material	4. Local control

Codes:

A) a-2, b-4, c-3, d-1
B) a-3, b-1, c-2, d-4
C) a-3, b-1, c-4, d-2
D) a-2, b-1, c-3, d-4

69. Let $X_1, X_2, \ldots, X_N$ be N vectors constituting a sample from $N_p(\mu, \Sigma)$, where $p < N$. If $\overline{X}$ and A denote the sample mean vector and the matrix of sum of squares and products of deviations about the mean, then maximum likelihood estimators of μ and Σ are respectively given by

A) $\overline{X}$ and $\frac{1}{N-1}A$
B) $\overline{X}$ and $\frac{1}{N}A$
C) $N\overline{X}$ and $\frac{1}{N}A$
D) $N\overline{X}$ and $\frac{1}{N-1}A$

70. Let X be a random vector which is assumed to be distributed as multivariate normal, $N_p(0, \Sigma)$. Then a necessary and sufficient condition for the quadratic forms $X'AX$ and $X'BX$ to be independently distributed is that

A) $AB = 0$
B) $AB \neq 0$
C) $A\Sigma B = 0$
D) $A\Sigma B \neq 0$

71. If $X_1, X_2, \ldots, X_N$ are N vectors which constitute a sample from $N_p(0, \Sigma)$ with Σ being non-singular, then the distribution of $N(\overline{X} - \mu)' \Sigma^{-1} (\overline{X} - \mu)$ is

A) $t^2_{(p)}$
B) $t^2_{(N)}$
C) $X^2_{(N)}$
D) $X^2_{(p)}$

72. Consider the test for $H_0 : \mu = \mu_0$, where μ is the mean vector of a multivariate normal $N_p(\mu, \Sigma)$ with unknown dispersion matrix Σ, based on a sample $X_1, X_2, \ldots, X_N$. If $\overline{X}$ and A denote the sample mean vector and the matrix of sum of squares and products of deviations about the mean, then the test statistic T^2 is given by

A) $N\,(\overline{X} - \mu_0)'\,A^{-1}(\overline{X} - \mu_0)$ B) $(N - 1)\,(\overline{X} - \mu_0)'\,A^{-1}(\overline{X} - \mu_0)$

C) $(\overline{X} - \mu_0)'\,A^{-1}(\overline{X} - \mu_0)$ D) $N(N - 1)(\overline{X} - \mu_0)'\,A^{-1}(\overline{X} - \mu_0)$

73. The transition probability matrix associated with a Markov chain is given by $P = \begin{bmatrix} 1/2 & 1/2 \\ 1/3 & 2/3 \end{bmatrix}$ and one of the eigen values of P is 1/6. Then the other eigen value is

A) $\frac{1}{6}$ B) -1 C) 1 D) 0

74. Consider the following statements regarding a Markov chain:
1. A Markov chain is irreducible if all states communicate with each other.
2. In a finite-state Markov chain all states can be transient.
Which of the above statements is/are true?

A) 1 only B) 2 only
C) Both 1 and 2 D) Neither 1 nor 2

75. For a Poisson process, the inter-arrival times are distributed as
A) i.i.d. gamma B) i.i.d. uniform
C) i.i.d. exponential D) i.i.d. Poisson

76. In a M/M/1 queue, suppose that customers arrive at a Poisson rate of one per every 12 minutes and that the service time is exponential at a rate of one service per 8 minutes. Then the average time a customer spends in the queue is
A) 16 minutes B) 24 minutes C) 12 minutes D) 20 minutes

77. Consider the model Y =T x S x C x I in time series analysis, where Y denotes the observation at time t and T, S, C, I represent the effects of the components trend, seasonal variation, cyclic variation and irregular variation respectively of a time series. Then which of the following is not true of the model?

A) It is known as the multiplicative model
B) It is the most commonly used model in time series analysis
C) The model assumes the independence of the four components of the time series
D) The model is used for prediction

78. Among the methods given below, which one is not used for measuring the component viz. seasonal variation of a time series?

A) Ratio to trend method
B) Simple average method
C) Moving average method
D) Ratio to moving average method

79. If Laspeyre's index number is 324 and Fisher's ideal index number is 216, then the Paasche's index number is:

A) 121
B) 144
C) 169
D) 196

80. The following are some of the adequacy tests for index numbers:

1. Time reversal test
2. Factor reversal test
3. Circular test

Then which of the above tests are satisfied by Fisher's ideal index number?

A) 1 and 2 only
B) 1 and 3 only
C) 2 and 3 only
D) 1, 2 and 3